Received: *May 17, 2001*, Accepted: *June 8, 2001*

Casimir energy and radius stabilization in five and six dimensional orbifolds

Eduardo Pontón and Erich Poppitz*

Department of Physics, Yale University
New Haven, CT 06520-8120, USA
E-mail: eduardo.ponton@yale.edu, erich.poppitz@yale.edu

Abstract: We compute the one-loop Casimir energy of gravity and matter fields, obeying various boundary conditions, in 5-dimensional $S^1/\mathbb{Z}_2$ and 6-dimensional $T^2/\mathbb{Z}_k$ orbifolds. We discuss the role of the Casimir energy in possible radius stabilization mechanisms and show that the presence of massive as well as massless fields can lead to minima with zero cosmological constant. In the 5-d orbifold, we also consider the case where kinetic terms localized at the fixed points are not small. We take into account their contribution to the Casimir energy and show that localized kinetic terms can also provide a mechanism for radius stabilization. We apply our results to a recently proposed 5-dimensional supersymmetric model of electroweak symmetry breaking and show that the Casimir energy with the minimal matter content is repulsive. Stabilizing the radius with zero cosmological constant requires, in this context, adding positive bulk cosmological constant and negative brane-tension counterterms.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Supersymmetry Breaking, Beyond Standard Model.

*Address after July 1, 2001: Department of Physics, University of Toronto, 60 St George St., Toronto, ON M5S 1A7, Canada

JHEP06(2001)019

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1. Introduction and summary

The idea that there are extra spatial dimensions in which gravity and, possibly, some or all matter fields can propagate has been the subject of renewed interest in the last few years. The Kaluza-Klein and "braneworld" ideas combined have been used to reformulate and address every conceivable problem of elementary particle physics [1]–[12].

One of the main theoretical issues in theories with extra dimensions is that of determining their size. In the absence of a stabilization mechanism, the Casimir energy tends to either inflate or contract the extra dimensions, as has been known since the work of refs. [13, 14]. The Casimir energy of various braneworld compactifications has also received attention recently [15]–[20].

In this paper, we focus on the Casimir energy and its role in radius stabilization in a particular class of Kaluza-Klein compactifications not discussed until recently

— field-theory orbifold compactifications.[1] Orbifolds are very useful tools for projecting out unwanted massless modes and/or breaking (super)symmetries. Several interesting phenomenological models using field theory orbifolds were proposed recently [22, 23].

In five dimensions, an orbifold was used to construct a supersymmetric model of (calculable!) electroweak symmetry breaking [22], predicting a light Higgs (related models were considered before in refs. [24, 25]). The realization of supersymmetry in this model is quite different from the usual minimal supersymmetric standard model — the theory below the compactification scale is not supersymmetric, while superpartners (the lightest being the stop) as well as mirror particles appear near that scale. In the six-dimensional case, a nonsupersymmetric $T^2/\mathbb{Z}_2$ orbifold construction was employed to build a higher-dimensional composite-Higgs model of electroweak breaking [23]. The issue of radius stabilization was not addressed in refs. [22, 23] — the Higgs-dependent part of the Casimir energy played a crucial role in the analysis of electroweak symmetry breaking in [22], but the radius was assumed to be fixed.

Originally, orbifolds were invented in string theory [26, 27] as a tool for constructing new consistent string backgrounds. Modular invariance of the worldsheet orbifold CFT and/or tadpole cancellation [28, 29, 30] place severe constraints on their consistency as fundamental theories. The field theory orbifolds that we discuss in this paper cannot, at least at present, be derived from known string constructions. Nevertheless, their success with electroweak symmetry breaking is appealing enough to warrant further study. We will only demand that the orbifold theories be consistent as low-energy effective theories (hence the requirement that the theory of the zero modes be anomaly free; for a recent discussion, see [31]), valid up to some energy cutoff scale. Consistency of the field theory description demands then that the cutoff be (at least) an order of magnitude or so larger than the inverse compactification length scale.

When performing loop calculations in the orbifold field theory, the orbifold boundary conditions lead to extra divergences at the orbifold fixed points, see, e.g. [32]. To cancel these, new terms localized at the fixed points have to be introduced in the lagrangian. These localized terms can be kinetic and mass terms, as well as interaction terms [33, 34]. Their coefficients are additional parameters of the orbifold theory. It is known [35], that these "brane-localized" terms can affect the spectrum of the theory and consequently the Casimir energy. It is then natural to expect that they can play a role in the mechanism of radius stabilization.

We will first work in an approximation neglecting the contributions of brane-localized kinetic and mass terms to the Casimir energy. This is a valid approximation

[1]We note that all field theory orbifolds discussed in this paper have some periodic fermions, hence the nonperturbative instability of the Kaluza-Klein vacuum towards decay into "nothing," pointed out in [21] and recently discussed in [15] is absent.

if the coefficients of brane-localized kinetic and mass terms are of the order of the loop-generated values (consistency and naturalness demand that the tree-level coefficients be at least as large as the loop-induced values [36]). We will then generalize the calculations to the case of larger brane-localized kinetic and mass terms.

In section 2.1, we describe the five dimensional $S^1/\mathbb{Z}_2$ setup. We then present, in section 2.2, a general discussion of the divergences in the one-loop Casimir energy, the counterterms required for their cancellation, and the contributions to the radius potential from the Casimir energy and counterterms. The discussion of section 2.2 applies equally well to the 6-d case.

Motivated by the 5-d and 6-d models, mentioned above, we calculate the Casimir energy in $S^1/\mathbb{Z}_2$, in section 2.3, and $T^2/\mathbb{Z}_k$ compactifications, in section 4. We compute the gravitational contribution, as well as those of even or odd massless matter fields obeying periodic or antiperiodic boundary conditions. In section 2.4, we also calculate the Casimir energy of fields with bulk mass terms in the 5-dimensional case and discuss a mechanism for stabilizing the radius with massive fields. The computations of sections 2.3, 2.4, and 4 are sufficiently general to be useful when considering radius stabilization in concrete five and six dimensional models.

In section 3, we include the contribution of the brane kinetic terms to the Casimir energy in the $S^1/\mathbb{Z}_2$ orbifold. We show that the brane kinetic terms, if they are sufficiently large, can lead to radius stabilization at a size bigger than the cutoff length scale. For example, a value consistent with "naive dimensional analysis" (NDA) can yield a radius several times larger than the cutoff length. NDA arguments [36] constrain the coefficients of brane-localized kinetic terms (with dimension of length) to be several times the cutoff length scale.

A much larger value of the brane-localized kinetic terms can yield a radius much bigger than the cutoff length scale. To investigate the viability of such a scenario as an effective low-energy theory, in section 3.1 we ask whether the theory has a consistent perturbative expansion if the brane kinetic terms are significantly larger than the cutoff length scale. We investigate in detail the properties of the Green's function with the brane kinetic terms included and find that the brane-localized divergences are weaker, while bulk divergences are as in the theory without the brane kinetic terms; we note that this is consistent with the results of refs. [35, 37, 38]. The analysis of this Section strengthens the case for having values for the brane kinetic terms larger than those implied by NDA.[2]

Finally, in section 5, we apply our results to the 5-d supersymmetric models of electroweak breaking [22] (see also [39]). We show that the Casimir energy contribution of the massless fields to the radius potential is repulsive (here we treat brane

[2]This does not imply that the NDA arguments are incorrect. The main assumption of NDA — that there is a single fundamental length scale — does not hold when the brane kinetic terms take values larger than the cutoff length; we are merely saying that such a scenario can yield a consistent low-energy theory.

kinetic terms as small). We show that by fine-tuning the 5-d cosmological constant and "brane tension" counterterms (as discussed in section 2.2), an acceptable minimum for the radius with vanishing cosmological constant can be achieved. The sign of the bulk cosmological term required to have such a minimum is positive, while the brane tensions need to be negative. It thus appears that the required bulk counterterm is not supersymmetric, at least in the simplest 5-d supergravities. This might present a problem, because a non-supersymmetric counterterm would introduce an additional source of supersymmetry breaking and potentially affect the predictions of the model. Thus, deciding whether this, or any other, stabilization mechanism is viable requires fully embedding the model in 5-d supergravity; this is a problem that we leave for future work.

We present details of the calculation of one of the Casimir sums in an appendix (other sums in the paper, where indicated, are computed similarly). We end with concluding remarks and a discussion of some open issues in section 6.

2. Casimir energy on orbifolds

In this section, we discuss the general issue of the Casimir energy on orbifolds, in the framework of the five dimensional $S^1/\mathbb{Z}_2$ example. We begin by introducing the setup and notation in section 2.1. In section 2.2, we discuss the limits of applicability of the Casimir energy calculation, the divergences, and the counterterms required for their cancellation. The coefficients of these counterterms can not be computed from the low-energy effective theory alone — they are to be treated as parameters of the theory and are fixed by imposing normalization conditions on the potential for the radius. Sections 2.3, 2.4 contain the results of the calculation of the Casimir energy contribution of the gravitational field as well as of various massless and massive fields. In section 2.4 we also discuss a possible stabilization mechanism using the Casimir energy of massive fields.

2.1 The five dimensional $S^1/\mathbb{Z}_2$ example.

The general parameterization of the interval of an S^1 compactification is as follows, see, e.g. [13]:

$$ds^2 = \phi^{-1/3} \left(g_{\mu\nu} + A_\mu A_\nu \phi\right) dx^\mu dx^\nu + 2\phi^{2/3} A_\mu dx^\mu dy + \phi^{2/3} dy^2 \,. \qquad (2.1)$$

Here the four-dimensional indices are denoted by μ and the five-dimensional coordinate y (taken to have dimension of length) is assumed to change between $-L$ and L (until the physical radius is fixed, the scale L is a completely arbitrary length scale). For a general fluctuating background the fields $g_{\mu\nu}$, A_μ, and ϕ can depend on x^ν and y. The $\mathbb{Z}_2$ orbifold is obtained after identifying points on the circle related by a reflection in the fifth coordinate, $y \simeq -y$. The invariance of the interval (2.1) under the $\mathbb{Z}_2$

symmetry determines the transformation properties of the fields: $g_{\mu\nu}(y) = g_{\mu\nu}(-y)$, $A_\mu(y) = -A_\mu(-y)$, and $\phi(y) = \phi(-y)$. Since the field A_μ is odd under the $\mathbb{Z}_2$ symmetry, it can not have a zero mode.

The parameterization of the metric in (2.1) is convenient, because in the four-dimensional effective field theory of the zero modes it gives rise to four-dimensional Einstein gravity with metric tensor $g_{\mu\nu}(x)$, coupled to a dilaton[3] $\phi(x)$ and, before the orbifold projection, an abelian gauge field $A_\mu(x)$. Since the $\mathbb{Z}_2$ orbifold forbids the appearance of a zero mode of A_μ, we omit this field in what follows. The zero-mode effective theory is valid below the scale of the mass of the lowest Kaluza-Klein excitation, i.e. at energies below $(\phi^{1/3}L)^{-1}$; note that the physical size of the extra dimension is $\phi^{1/3}L$. More precisely, the five dimensional Einstein action, evaluated on a background (2.1) with the fields $g_{\mu\nu}(x), \phi(x)$, dependent only on x^μ is given by:

$$M_5^3 \int d^5x \sqrt{G} R(G) = M_4^2 \int d^4x \sqrt{g} \left[R(g) + \frac{1}{6} \frac{\partial_\mu \phi \partial_\nu \phi g^{\mu\nu}}{\phi^2} \right]. \tag{2.2}$$

In the above formula G denotes the five dimensional metric tensor that can be read off eq. (2.1), with $A_\mu = 0$ and the rest of the fields only dependent on x^μ, while the four and five dimensional Planck scales are related by $M_4^2 = LM_5^3$; in the orbifold theory we only integrate over the fundamental region $0 \leq y \leq L$.[4]

2.2 Casimir energy, divergences, and counterterms

Our goal is to study the generation of a potential $V(\phi)$ — and the possible existence of a minimum — for the dilaton field ϕ due to quantum effects of the nonzero modes of the fields g, A, ϕ, as well as to quantum contributions to the Casimir energy of matter fields (massless or not) that might be present.

We will perform the calculation of the Casimir energy around a constant background $g_{\mu\nu} = \eta_{\mu\nu}$, $A_\mu = 0$, $\phi = \text{const}$, with $\eta_{\mu\nu}$ — the Minkowski metric. Dynamical issues, such as the time evolution of the background as a backreaction to the Casimir energy, can then only be studied for time intervals such that the deviation of the metric from the assumed constant background is small. In this paper, we are interested in the existence of static stable minima with vanishing four-dimensional cosmological constant. Other interesting issues, such as the cosmological evolution of the background, are left for future work.

We do not attempt to say anything about the cosmological constant problem here. To achieve vanishing cosmological constant at the minimum, we will resort to the fact that the computation of the Casimir energy is plagued with divergences, whose cancellation requires adding counterterms to the action (2.2). Divergences

[3] We will occasionally call this field a 'radion.'

[4] To avoid confusion, we note that the relation between four dimensional and five dimensional Planck scales involves the physical size of the orbifold, $\phi^{1/3}L$, instead of the arbitrary scale L; however, the form (2.2) is more convenient before fixing $\langle\phi\rangle$.

are short-distance phenomena and the counterterms needed to cancel them are local terms, respecting the short-distance symmetries of the theory. The Casimir energy, on the other hand, is a global effect, depending on the topology and boundary conditions of the compactification, and can not be described by a local term preserving the short-distance symmetries. For example, in the S^1 compactification, (bulk) counterterms (using a generally covariant regulator) should respect 5-d general covariance and should not depend on the fact that at large distances it is broken by the compactification. For the constant flat metric background of interest to us, there is only one divergent counterterm—the 5-d cosmological constant term (this is, strictly speaking, true only in the unorbifolded case, see discussion below). This term is:

$$\alpha \int d^5x \sqrt{G} = \alpha L \int d^4x\, \phi^{-1/3} \,, \tag{2.3}$$

and thus contributes a potential $\sim \phi^{-1/3}$ to the four-dimensional effective action (2.2). We will treat its coefficient α (of dimension $(\text{mass})^5$) as a parameter to be fixed by the normalization conditions of the potential:

$$V(\phi) = V'(\phi) = 0 \,. \tag{2.4}$$

We note that for values of ϕ, such that eqs. (2.4) hold, the metric background is flat and thus the calculation of the Casimir energy — which assumed that — is self-consistent.

It is clear that if only massless fields are present and the only scale in the problem is M_5, the potential for ϕ due to the massless fields' Casimir energy is monotonic in ϕ and minimized either at $\phi = 0$ or ∞, depending on the matter content of the theory. We will see that the massless fields' Casimir contribution to the effective action (2.2) is proportional to:

$$\frac{1}{L^4} \int d^4x\, \phi^{-2} \,. \tag{2.5}$$

If the potential (2.5) is minimized for $\phi \to 0$, one has to resort to strong curvatures or other effects beyond the reach of the effective theory to stabilize the radius and the effective theory description of the low-energy physics is not valid. If, on the other hand, the minimum is at $\phi \to \infty$ one has a typical runaway potential (exponential, if the kinetic terms are rendered canonical by a field redefinition). This potential may provide for an acceptable form of quintessence. Our calculations show that in the five-dimensional models with the supersymmetric standard model in the bulk (with the minimal matter content) it is the second possibility that is realized.

Adding the cosmological constant counterterm $\sim \phi^{-1/3}$, eq. (2.3), changes the shape of the potential (2.5) and can generate a minimum. However, it is impossible to have a vanishing cosmological constant at the minimum with only the two contributions (2.5) and (2.3) to the potential. The nonzero cosmological constant at the minimum renders the calculation inconsistent.

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Clearly, the resolution is to introduce another scale in the problem. In what follows we consider the possible ways to do this.

The first, "nonminimal," way is to introduce additional fields in the theory and is applicable equally well to the unorbifolded theory. One can, for example, include a field with mass smaller than the cutoff of the five dimensional theory. In section 2.4, we calculate the Casimir energy of massive fields and show that upon adding the three contributions — massless, massive, and cosmological constant counterterm — it is possible to achieve a stable minimum for ϕ with vanishing cosmological constant, thus rendering the calculation self-consistent. The physical size of the extra dimension then is typically of order the inverse mass of the field. However, we will see that this mechanism, though in principle viable, can not be implemented in realistic models with the (supersymmetric) standard model fields in the five-dimensional bulk.

A second possibility — we call it "minimal," since it does not involve introducing new fields solely for the purpose of radius stabilization — is to explore the fact that we are compactifying not on a smooth circle, but on an orbifold. The Green functions $G(x - x'; y, y')$ of fields with orbifold boundary conditions have, in addition to the usual short-distance singularity when $x^\mu \to x^{\mu\prime}$, $y \to y'$, a singularity localized at the fixed points of the orbifold $y = y' = 0, L$ (see for example ref. [32], where the Green function of a scalar field in four dimensions with Dirichlet boundary conditions at a fixed two-plane is discussed). These singularities lead to divergent terms localized at the fixed points; for recent discussions see refs. [33, 34].

For example, since the Casimir energy is the expectation value of the energy momentum tensor, related to the Green function (e.g. of a scalar field) by:

$$\langle T_{MN}(X)\rangle = (\partial_M \partial'_N + \cdots) G(X, X')|_{X \to X'} \,, \tag{2.6}$$

(here X denotes both x^μ and y), the singularities of $G(X, X')$ at the orbifold fixed points lead to divergent contributions to the Casimir energy localized at the fixed points, in addition to the 5-d bulk cosmological constant term mentioned above. Canceling these extra divergences requires adding counterterms localized at the fixed points (i.e. "brane tensions"). We will treat the coefficients of these terms as parameters, which can not be determined in the low-energy theory. (We have to assume that the "brane tensions" are small enough in order to ignore the warping they typically produce; we will comment on this in our discussion in section 5.) For the metric background (2.1), these localized terms on the $S^1/\mathbb{Z}_2$ orbifold are of the form:

$$\beta \int d^5x \delta(y) \sqrt{\tilde{g}} = \beta \int d^4x \, \phi^{-2/3} \,, \tag{2.7}$$

where $\tilde{g}$ denotes the induced metric at $y = 0$, and a similar term at the $y = L$ fixed point. Thus the localized terms scale differently from the terms due to the cosmological constant counterterm (2.3) and the massless fields' "bulk" Casimir energy (2.5).

We will show that in some cases it is possible to achieve a (local) minimum of $V(\phi)$ with vanishing cosmological constant. The physical size of the extra dimension is then set by the coefficients of the tension terms.

The singularity of the Green function leads also to other divergences, for example to kinetic terms localized at the orbifold fixed points [33]; these have been considered recently from a somewhat different perspective in refs. [35, 37, 38]. The localized kinetic terms have the form:

$$\int d^5x \, c \, \delta(y) \, \partial_\mu \Phi \partial^\mu \Phi \tag{2.8}$$

and introduce a new length scale, c, in the problem. The localized kinetic terms can significantly change the spectrum of the nonzero modes [35], and hence the ϕ-dependence of the Casimir energy. We calculate, in section 3, the Casimir energy in the theory with the inclusion of such terms and find that it is possible to achieve a minimum for ϕ with vanishing cosmological constant; we also discuss localized mass terms.

Finally, even though we will not pursue it here, we should mention the possibility to combine the "minimal" and "nonminimal" approaches, exploring the fact that the compact space is an orbifold and at the same time introducing new fields for the purpose of size stabilization. This is much in the spirit of mechanisms considered before in codimension one and two in refs. [40, 41]. These are classical mechanisms: one postulates that the fixed points are sources for some bulk field, e.g., a scalar field. In codimension two, see [41], one introduces two kinds of massless fields, "dual" to each other, and finds that the energy of one type grows with size while that of the other decreases (to achieve that, one has to impose certain boundary conditions at the fixed points), leading to a minimum for the size. Other "nonminimal" mechanisms have been discussed in [42, 43]. We note that if one does not want to achieve a hierarchically large radius, it is possible to use a classical mechanism of the type discussed above to balance a repulsive Casimir force.

2.3 Gravity and massless matter fields

In this section, we calculate the Casimir energy due to the gravitational field and to other massless fields, with both periodic and antiperiodic boundary conditions on the S^1. Here we assume that the contributions of the various terms at the fixed points can be neglected, while section 3 contains a discussion of the more general case.

Most of the discussion in this section is not new. For completeness, we briefly review the calculation of ref. [13] of the contribution to the Casimir energy of the fields $g_{\mu\nu}, A_\mu, \phi$. We then generalize the calculation to include matter fields with both periodic and antiperiodic boundary conditions on the S^1. Finally, we give the generalization to the orbifold case. We will see that, if the contributions from fixed-point-localized terms are negligible, the Casimir energy of the $S^1/\mathbb{Z}_2$ orbifold is one half that of the S^1 compactification.

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Later on, in section 5, we apply the formulae obtained for various fields to calculate the potential for ϕ generated in the models of ref. [22], where the supersymmetric standard model lives in the five dimensional bulk. We show that in these models the Casimir potential for ϕ is repulsive, hence (for vanishing coefficients of the counterterms) the compact space tends to expand to infinite size.

We begin with the Casimir energy of the gravity sector. As in ref. [13], only the nonzero modes contribute to the potential for ϕ, and the contribution of the gravitational multiplet equals that of five massless real scalar fields. This can be seen after appropriate gauge fixing and is essentially due to the fact that the five-dimensional graviton has five polarization states [13]. Thus, it is enough to compute the vacuum energy of a single massless scalar field, with periodic boundary conditions along the S^1:

$$\begin{aligned} V^{+,\,\mathrm{scalar}} &= \frac{1}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\log\left(k^2+\frac{\pi^2 n^2}{\phi\, L^2}\right) \\ &\equiv -\frac{d}{ds}\zeta^{+,\,\mathrm{scalar}}(s)\big|_{s=0}\,. \end{aligned} \tag{2.9}$$

The second line indicates that we use ζ-function regularization to calculate the Casimir energy. Infinite contributions to the Casimir energy are thrown out by the regularization and counterterms have to be added by hand, as discussed above. The periodic scalar ζ-function is defined as:

$$\begin{aligned} \zeta^{+,\,\mathrm{scalar}}(s) &= \frac{1}{2}\sum_{n=-\infty}^{\infty}\int\frac{d^4k}{(2\pi)^4}\left(k^2+\frac{\pi^2 n^2}{\phi\, L^2}\right)^{-s} \\ &= \frac{1}{16\pi^2}\,\frac{1}{(2-s)(1-s)}\,\frac{\pi^{4-2s}}{L^{4-2s}\phi^{2-s}}\zeta(2s-4)\,, \end{aligned} \tag{2.10}$$

with $\zeta(s)$ — the Riemann zeta function. Plugging into eq. (2.9) yields for the massless periodic scalar contribution to the potential in eq. (2.2):

$$V^{+,\,\mathrm{scalar}}(\phi) = \frac{\pi^2}{16}\frac{-\zeta'(-4)}{\phi^2 L^4} = -\frac{3\zeta(5)}{64\pi^2}\,\frac{1}{\phi^2 L^4}\,, \tag{2.11}$$

which is the result of [13]. We thus find that the contribution of the gravitational fluctuations to the Casimir energy, equal to $5V^{+,\,\mathrm{scalar}}(\phi)$, is attractive ($\zeta(5)\simeq 1.034$), i.e. the circle tends to shrink to zero size.

Having in our disposal the result for the periodic massless scalar, we can easily enumerate the Casimir contributions of all other periodic massless fields, e.g. by using knowledge about five-dimensional supersymmetry multiplets. Below, we summarize the results for all massless periodic fields of interest:

$$\begin{aligned} V^{+,\,\mathrm{graviton}}(\phi) &= 5\,V^{+,\,\mathrm{scalar}}(\phi)\,, \\ V^{+,\,\mathrm{fermion}}(\phi) &= -4\,V^{+,\,\mathrm{scalar}}(\phi)\,, \\ V^{+,\,\mathrm{vector}}(\phi) &= 3\,V^{+,\,\mathrm{scalar}}(\phi)\,, \\ V^{+,\,\mathrm{gravitino}}(\phi) &= -8\,V^{+,\,\mathrm{scalar}}(\phi)\,. \end{aligned} \tag{2.12}$$

It is also of interest to compute the contribution of matter fields (hypermultiplets) with antiperiodic boundary conditions on the S^1. The scalar contribution $V^{-,\,\mathrm{scalar}}(\phi)$ is given by the same expression as (2.9) except for the sum being over half-integers. Thus, the formula for the antiperiodic zeta function (2.10) remains the same safe for the replacement $\zeta(2s-4) \to \zeta(2s-4, 1/2)$, where $\zeta(s, 1/2) = (2^s - 1)\zeta(s)$ is the generalized zeta function. Evaluating the derivative, we obtain a repulsive potential for ϕ from an antiperiodic real scalar field:

$$V^{-,\,\mathrm{scalar}}(\phi) = -\frac{15}{16}\, V^{+,\,\mathrm{scalar}}(\phi)\,, \tag{2.13}$$

while the contribution of an antiperiodic fermion is attractive:

$$V^{-,\,\mathrm{fermion}}(\phi) = \frac{15}{4}\, V^{+,\,\mathrm{scalar}}(\phi)\,. \tag{2.14}$$

Finally, we come to the $S^1/\mathbb{Z}_2$ orbifold Casimir energy. Neglecting the effects of terms localized at the fixed points, the orbifold amounts to simply throwing out all modes even under $y \to -y$ (for odd fields) or odd (for even fields). Since only the non-zero modes contribute to the Casimir energy, it is easy to see that in each case this amounts to throwing out half the modes in the sum over n in (2.9). (We note that an additional factor of $1/2$ will occur because in the orbifold we only integrate from $y = 0$ to $y = L$. This, however, will be irrelevant for us.)

As noted in the introduction, including the contribution to the Casimir energy of the massless fields alone (without including brane tensions) it is not possible to obtain a minimum for ϕ with vanishing cosmological constant. We will, in what follows, use the results from this section to discuss ways of obtaining a minimum when other possible contributions are also included.

2.4 Casimir energy and radius stabilization with massive matter fields.

In this section, we calculate the contribution of both periodic and antiperiodic massive fields to the Casimir energy. We begin with the periodic case, when the vacuum energy for a real scalar of mass μ is given by:

$$V^{+,\,\mathrm{scalar}}(\phi,\mu) = \frac{1}{2}\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{(2\pi)^4}\log\left(k^2 + \frac{\pi^2 n^2}{\phi\, L^2} + \frac{\mu^2}{\phi^{1/3}}\right) \equiv -\frac{d}{ds}\zeta_\mu^{+,\,\mathrm{scalar}}(s)|_{s=0}\,. \tag{2.15}$$

The massive periodic scalar ζ-function is:

$$\begin{aligned}\zeta_\mu^{+,\,\mathrm{scalar}}(s) &= \frac{1}{2}\sum_{n=-\infty}^{\infty}\int \frac{d^4k}{(2\pi)^4}\left(k^2 + \frac{\pi^2 n^2}{\phi\, L^2} + \frac{\mu^2}{\phi^{1/3}}\right)^{-s} \\ &= \frac{1}{32\pi^2}\frac{1}{(2-s)(1-s)}\frac{\pi^{4-2s}}{L^{4-2s}\phi^{2-s}}\, F\left(s-2; 0, \frac{\mu L \phi^{1/3}}{\pi}\right),\end{aligned} \tag{2.16}$$

where $F(s;0,c)$ is the series:

$$F(s;0,c) = \sum_{n=-\infty}^{\infty} \left(n^2 + c^2\right)^{-s} , \tag{2.17}$$

and can be evaluated as described in the appendix. The contribution of an antiperiodic scalar field can also be evaluated — the only change is that the sum in (2.17) is over half integers.

The results for the Casimir contribution $V^{+,\mathrm{scalar}}(\phi,\mu)$, $V^{-,\mathrm{scalar}}(\phi,\mu)$ of periodic or antiperiodic massive scalars respectively — omitting both finite and infinite contributions to the bulk cosmological constant — can be written in a common form:

$$\begin{aligned} V^{\pm,\mathrm{scalar}}(\phi,\mu) = -\frac{3}{64\pi^2}\frac{1}{L^4\phi^2}\Big[&\mathrm{Li}_5\left(\pm e^{-2L\mu\phi^{1/3}}\right) + 2L\mu\phi^{1/3}\mathrm{Li}_4\left(\pm e^{-2L\mu\phi^{1/3}}\right) + \\ &+\frac{4}{3}L^2\mu^2\phi^{2/3}\mathrm{Li}_3\left(\pm e^{-2L\mu\phi^{1/3}}\right)\Big] , \end{aligned} \tag{2.18}$$

where $\mathrm{Li}_n(x)$ are the polylogarithm functions. Since $\mathrm{Li}_5(1) = \zeta(5)$ and $\mathrm{Li}_5(-1) = -15\zeta(5)/16$, the above expression reduces to the massless formulae for the periodic (2.11) and antiperiodic (2.13) case in the small mass or small radius ($\phi \to 0$) limit. As in the massless case, the contribution of a massive fermion on the S^1 is equal to minus 4 times the real scalar contribution; the $\mathbb{Z}_2$ orbifold contribution is one-half the unorbifolded contribution in each case.[5]

The bulk mass μ introduces a new scale in the problem, leading one to expect that stabilizing the radius using the Casimir energy of massive fields should be possible in certain cases. To see what the necessary conditions are, consider the following toy model with a single scale μ. Let there be a periodic field (scalar or fermion; or a number of fields) of mass μ and a massless sector (consisting of gravity and possibly other massless fields). Introducing the variable $x = 2\mu L\phi^{1/3}$, we can write the total potential for x in the following form:

$$\begin{aligned} V(x) &= \frac{|\mathrm{const}|}{x^6}\left(\alpha x^5 + \beta\left(\mathrm{Li}_5(e^{-x}) + x\mathrm{Li}_4(e^{-x}) + \frac{x^2}{3}\mathrm{Li}_3(e^{-x})\right) + \gamma\right) \\ &\equiv x^{-6} f(x) , \end{aligned} \tag{2.19}$$

where the first term is the cosmological constant counterterm, while the second and third terms are the contributions to the Casimir energy of the masssive and massless fields, respectively. That the potential $V(x)$ can have a stable minimum is easy to understand qualitatively. A stable minimum can occur only[6] if the massive contribution dominates near the origin (and is repulsive), while the massless term

[5] The potential eq. (2.18) was independently derived in ref. [24].

[6] It is important to note that the term multiplying β is a monotonically decreasing function of x equal to $Li_5(1)$ at $x = 0$.

(with opposite sign) takes over at larger distances, where the massive contribution is exponentially suppressed. The normalization conditions (2.4) become, for the potential $V(x)$ (2.19):

$$f'(x) = 0\,, \qquad f(x) = 0\,, \tag{2.20}$$

and can be shown to have solutions provided the coefficients γ and β have opposite signs, while α and β have the same sign; in addition, the solution of (2.20) is a minimum if $\gamma < 0$. One also requires that $|\gamma| < \mathrm{Li}_5(1)|\beta|$ so that the massive repulsive contribution dominates near $x = 0$ ensuring the existence of a minimum.

The conclusion one can draw from this simple one-scale model is that it is possible to have a stable minimum with zero cosmological constant, provided: i) the massless contribution is attractive, ii) the massive contribution is repulsive and dominant at small radii, and, iii) the bulk cosmological constant is fine-tuned (and positive, corresponding to $\alpha > 0$); as usual, there is also a runaway minimum at $x \to \infty$.[7] Barring unnaturally large coefficients, the radius is stabilized at a size of order the inverse mass of the field, which is the only length scale in our toy model.

Using the results of this section, it is straightforward to include fields of different masses, as well as brane tension counterterms in the discussion of radion stabilization.

3. Casimir energy contribution of brane-localized kinetic terms

In this section, we calculate the contribution to the Casimir energy of a massless scalar field while taking into account the presence of kinetic terms localized at the fixed points $y = 0, L$. These terms can significantly affect the Kaluza-Klein spectrum of the field and change the Casimir energy. Readers interested only in the answer can skip to the final result, eq. (3.15), and the following discussion of the radius dependence of the Casimir energy with boundary terms.

In the metric background (2.1) with $g_{\mu\nu} = \eta_{\mu\nu}$, $A_\mu = 0$, $\phi = const.$, we take the quadratic action of a real periodic scalar field Ψ to be:[8]

$$\frac{1}{2}\int d^4x \int_{-L}^{L} dy \left[\partial_\mu\Psi\partial^\mu\Psi + \frac{1}{\phi}(\partial_y\Psi)^2 + \left(2c_0\phi^{-1/3}\delta(y) + 2c_L\phi^{-1/3}\delta(L-y)\right)\partial_\mu\Psi\partial^\mu\Psi\right], \tag{3.1}$$

[7] A similar conclusion can be drawn if one considers instead an antiperiodic massive field — once again one requires that its contribution to the Casimir energy be repulsive (hence the massive field is an antiperiodic scalar) and dominant at short distances in order to achieve a minimum (in this case $\alpha > 0$ as well).

[8] Derivatives w.r.t. y will also generally appear. These require a more careful treatment in the thin wall limit we are studying here. For simplicity, we consider only the terms of eq. (3.1). These should be sufficient to illustrate the effects of localized terms on the Casimir energy.

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where c_0 and c_L are the two length scales introduced by the localized kinetic terms. Expanding the field $\Psi(x, y)$ into Fourier components $\Psi_k(y)$ of four-dimensional mass $\sqrt{k^2}$, we find the equation of motion:

$$\left[\partial_y^2 + \phi k^2 + (2c_0\phi^{-1/3}\delta(y) + 2c_L\phi^{-1/3}\delta(L-y))\phi k^2\right]\Psi_k(y) = 0\,. \qquad (3.2)$$

Now we introduce the variables $\tilde{k}^2 \equiv \phi k^2$ and $\tilde{c}_{0,L} \equiv \phi^{-1/3}c_{0,L}$, and solve eq. (3.2) for even periodic fields $\Psi(y+2L) = \Psi(y), \Psi(-y) = \Psi(y)$. We find that the values of $\tilde{k}$ for which the solution has the appropriate discontinuities to match the delta functions are determined by the solutions of:

$$\tan(\tilde{k}L) = -\frac{\tilde{k}(\tilde{c}_0 + \tilde{c}_L)}{1 - \tilde{c}_0\tilde{c}_L\tilde{k}^2}\,. \qquad (3.3)$$

When $c_{0,L} \to 0$, the equation simply gives the masses of the Kaluza-Klein modes $|k_n| = |n|\pi/L$. It is clear that the solutions of eq. (3.3) are also labeled by an integer, and we shall denote them by $\tilde{k}_n$. Note also that if $c_{0,L}$ are positive, there are no negative k^2 solutions, i.e. no tachyons.

Expanding the field $\Psi(x, y) = \sum_{n=0}^{\infty}\varphi_n(x)\Psi_{k_n}(y)$ in terms of the (even) modes $\Psi_{k_n}(y)$, we find that the quadratic part of the four dimensional theory of the "Kaluza-Klein" modes $\varphi_n(x)$ is governed by the action:

$$\frac{1}{2}\int d^4x \sum_{m,n=0}^{\infty}\left(\partial_\mu\varphi_n\partial^\mu\varphi_m + \frac{\tilde{k}_n^2}{\phi}\varphi_n\varphi_m\right)\gamma_{mn}\,. \qquad (3.4)$$

The normalization coefficients obey:

$$\gamma_{mn} = \int_{-L}^{L} dy\Psi_{k_n}(y)\Psi_{k_m}(y)\left(1 + 2\tilde{c}_0\delta(y) + 2\tilde{c}_L\delta(y-L)\right) = 0 \quad \text{for} \quad m \neq n\,. \qquad (3.5)$$

The orthogonality of the Kaluza-Klein wavefunctions follows from the "Schrödinger" equation (3.3). The diagonal normalization coefficients γ_{nn} depend on n, however, in contrast to conventional Kaluza-Klein compactifications; this is important when considering interactions involving the Kaluza-Klein modes [37, 38]. The normalization coefficients drop out of the Casimir energy, however.

The Casimir energy of the real scalar field, even under the $\mathbb{Z}_2$ orbifold projection, can be then written as follows:

$$V^{+,\text{scalar}}(\phi; c_{0,L}) = \frac{1}{2}\sum_{n=0}^{\infty}\int\frac{d^4p}{2\pi^4}\log\left(p^2 + \frac{\tilde{k}_n^2}{\phi}\right) \equiv -\frac{d}{ds}\zeta_{c_{0,L}}(s)\Big|_{s\to 0}\,. \qquad (3.6)$$

The ζ-function is that of the operator in (3.2) and is given by:

$$\zeta_{c_{0,L}}(s) = \frac{1}{32\pi^2}\frac{1}{(2-s)(1-s)}\frac{1}{L^{4-2s}\phi^{2-s}}F(2s-4)\,, \qquad \text{where } F(s) \equiv \sum_{n=1}^{\infty}\frac{1}{x_n^s}\,, \quad (3.7)$$

where the sum is over the nonnegative roots of eq. (3.3), which written in terms of $x = \tilde{k}L$, has the form $\tan x = -xa/(1-bx^2)$. The coefficients a, b are expressed in terms of the ratios of the length scales $c_{0,L}$ from (3.1) to the physical size of the orbifold $l_{\rm phys} = L\phi^{1/3}$ as follows:

$$
\begin{aligned}
a &= \frac{\tilde{c}_0 + \tilde{c}_L}{L} = \frac{c_0 + c_L}{l_{\rm phys}}\,, \\
b &= \frac{\tilde{c}_0 \tilde{c}_L}{L^2} = \frac{c_0 c_L}{l_{\rm phys}^2}\,.
\end{aligned}
\tag{3.8}
$$

The sum $F(s)$ of eq. (3.7) can be taken by means of contour integrals as follows. Introduce the function:

$$
f(z) = az + (1 - bz^2)\tan z\,, \tag{3.9}
$$

and consider the contour integral:

$$
I(s) = \frac{1}{2\pi i}\int_C dz \frac{1}{z^s}\frac{f'(z)}{f(z)}\,, \qquad C = C_\infty + C_+ + C_\epsilon + C_-\,, \tag{3.10}
$$

where the contour C_∞ is an infinite semicircle in the Re$z > 0$ half-plane, C_+ runs along the imaginary axis from $z = i\infty$ to $z = i\epsilon$, C_- — from $z = -i\epsilon$ to $-i\infty$, and C_ϵ is a small semicircle from $z = i\epsilon$ to $z = -i\epsilon$ in the Re$z > 0$ half-plane. The sum $F(s)$ in eq. (3.7) can be written as:

$$
F(s) \equiv \sum_{n>0}\frac{1}{x_n^s} = I(s) + \frac{1}{\pi^s}\sum_{n\geq 0}\frac{1}{\left(n+\frac{1}{2}\right)^s} = I(s) + \frac{2^s - 1}{\pi^s}\zeta(s)\,, \tag{3.11}
$$

where we used the fact that $I(s)$ is determined by the poles of f'/f inside the contour C, which occur at the zeros ($z = x_n$) and poles ($z = (n+1/2)\pi$) of $f(z)$, eq. (3.9).

We are interested in computing $F'(-4)$, which, as usual is defined by analytic continuation. The integral over C_∞ vanishes for a sufficiently large positive Re(s). The rest of the integral can be explicitly evaluated and written as follows:

$$
\begin{aligned}
I(s) = \; & \frac{s}{\pi}\sin\left(\frac{\pi s}{2}\right)\int_\epsilon^\infty dy\, y^{-s-1}\log\frac{ay + (1+by^2)\tanh y}{1 + ay + by^2} + \\
& + \frac{s}{\pi}\sin\left(\frac{\pi s}{2}\right)\int_\epsilon^\infty dy\, y^{-s-1}\log(1 + ay + by^2) - \\
& - \frac{s\epsilon^{-s}}{2\pi}\int_{-\pi/2}^{\pi/2} d\theta\, e^{-is\theta}\log f(\epsilon e^{i\theta}) - \frac{\epsilon^{-s}}{2}\cos\left(\frac{\pi s}{2}\right).
\end{aligned}
\tag{3.12}
$$

It is easy to see from (3.12) that the divergences as $\epsilon \to 0$ cancel. This form of the integral is also appropriate for analytic continuation to negative s. The first integral is well-behaved for $s < 0$. The second integral can be explicitly evaluated in terms of hypergeometric functions; its analytic continuation for negative s vanishes as $\epsilon \to 0$. The analytic continuation of the last line for $s < 0$ also vanishes in this limit. Thus, for negative s, we only have the first integral, where the limit $\epsilon \to 0$ can be taken safely and we arrive at our formula for the analytic continuation of $F(s)$ to negative s:

$$F(s) = \frac{2^s - 1}{\pi^s}\zeta(s) + \frac{s}{\pi}\sin\left(\frac{\pi s}{2}\right)\int\limits_0^\infty dy\, y^{-s-1}\log\frac{ay + (1+by^2)\mathrm{tanh}y}{1 + ay + by^2}\,, \tag{3.13}$$

which gives rise to:

$$-\frac{d}{ds}F(2s-4)\Big|_{s=0} = \frac{15\,\pi^4}{8}\,\zeta'(-4) + 4\int\limits_0^\infty dy\, y^3\log\frac{ay + (1+by^2)\mathrm{tanh}y}{1 + ay + by^2}\,. \tag{3.14}$$

In the limit $a = b = 0$, the expression (3.14), upon substitution in (3.6), yields the Casimir energy (2.11) without the boundary terms.

The final result for the Casimir energy of the even periodic scalar with boundary kinetic terms is, therefore:

$$V^{+,\,\mathrm{scalar}}(\phi; c_{0,L}) = \frac{1}{64\pi^2 L^4\phi^2}\left[\frac{15}{8}\pi^4\zeta'(-4) + \rho\left(\frac{c_0 + c_L}{L\phi^{1/3}}, \frac{c_0 c_L}{L^2\phi^{2/3}}\right)\right], \tag{3.15}$$

with $\rho(a,b) = 4\int\limits_0^\infty dyy^3\log\left[(ay + (1+by^2)\mathrm{tanh}y)/(1 + ay + by^2)\right]$. While the form of the ϕ-dependence is rather complicated and can be studied in detail only numerically, eq. (3.15) can be used to illustrate some qualitative features of the Casimir energy with boundary terms.

Consider first the large-radius behavior — the limit $\phi \to \infty$. In this limit, the coefficients a, b vanish and eq. (3.15) reduces to (2.11) — except for a factor of 1/2 due to the orbifold — giving thus an attractive potential at large ϕ:

$$V^{+,\,\mathrm{scalar}}(\phi \to \infty; c_{0,L}) \sim -\frac{3\zeta(5)}{128\pi^2}\frac{1}{\phi^2 L^4} + \cdots\,. \tag{3.16}$$

Thus, as expected, the boundary terms do not affect the large-radius behavior of the Casimir energy.

The limit of small radii $\phi \to 0$ is more involved. To study it, note that the function ρ of (3.15) can be written as:

$$\rho = 4\phi^{4/3}\int\limits_0^\infty dyy^3\log\left[\frac{y + (1+\hat{b}y^2)\mathrm{tanh}\phi^{1/3}y}{1 + y + \hat{b}y^2}\right], \tag{3.17}$$

where the hat indicates that the ϕ dependence has been scaled out of b; in addition, to simplify our formulae, we have chosen the arbitrary length scale L such that $a \equiv 1$.

In the limit when the boundary term at one of the fixed points vanishes, i.e. $\hat{b} = 0$, it is easy to see that $\rho \to 0$ as $\phi \to 0$ (the integral in (3.17) diverges as $\phi \to 0$, but more slowly than the $\phi^{4/3}$ prefactor, such that the whole expression (3.17) vanishes). The small-ϕ potential is therefore repulsive. Thus, in the small radius limit the Casimir energy for the periodic scalar with boundary term at only one of the fixed points is repulsive, while the large-radius behavior, eq. (3.16), is attractive. We conclude that in the limiting case when the boundary kinetic terms at one of the fixed points, say $c_L = 0$, vanishes, the Casimir energy exhibits a minimum for ϕ. The minimum occurs for values of the physical radius of order the length scale c_0 set by the nonvanishing boundary kinetic term.

The behavior of the Casimir energy with one nonzero boundary term can be inferred already from the eigenvalue equation (3.3) in the limit when one of the boundary terms vanishes — then the lowest eigenvalues are approximately the periodic Kaluza-Klein modes $n\pi/L$, while the large eigenvalues are the antiperiodic ones, i.e. $(n + 1/2)\pi/L$. One expects that the large-radius limit of the Casimir energy is dominated by the lowest eigenvalues — hence the attractive behavior characteristic of a periodic scalar — while the small-radius limit depends on the large eigenvalues, leading one to expect a repulsive behavior, as for an antiperiodic scalar. The behavior of the Casimir energy (3.6) in the two limits $\phi \to \infty$ and $\phi \to 0$ confirms this expectation.

In the case where both boundary terms are present, the analysis requires more care. Upon analyzing the integral (3.17) one finds that when $a \neq 0, b \neq 0$ the Casimir energy is attractive at small ϕ as well (as in the case without boundary terms). Thus, the existence or not of a minimum depends on relative strength of a and b. By comparing to the $a \neq 0$, $b = 0$ case, one expects that for a sufficiently small ratio of b/a, a local minimum will still persist. A numerical analysis confirms this expectation — it is sufficient to have $c_0/c_L \sim .3$ in order for a (local) minimum to exist. There is no minimum if $c_0 = c_L$, however.

Thus, we conclude that the radius can be stabilized by the boundary kinetic terms, provided there is some asymmetry between the fixed points.[9] As usual, the cosmological constant at the minimum can be adjusted to zero by tuning the coefficients of the counterterms. Finally, while for a periodic scalar with asymmetric boundary kinetic terms the minimum is local, we expect that for a periodic fermion the boundary kinetic terms will yield a global minimum.

To conclude this section, we consider the effect of brane kinetic terms on fields that obey antiperiodic boundary conditions. Solving the scalar equation (3.2) but

[9]We note that the classical mechanisms for radius stabilization of [40, 41], also rely on an assymetry between the fixed points ("branes").

imposing antiperiodic boundary conditions:

$$\Psi_k(y+2L) = -\Psi_k(y)\,, \tag{3.18}$$

we find that the modes can be divided in two types: i) modes that are even about $y = L$ with a spectrum determined by:

$$\tan(\tilde{k}L) = \frac{1}{\tilde{c}_L \tilde{k}}\,, \tag{3.19}$$

and ii) modes that are odd about $y = L$ with a spectrum determined by:

$$\tan(\tilde{k}L) = \frac{1}{\tilde{c}_0 \tilde{k}}\,. \tag{3.20}$$

The Casimir energy of such modes is calculated in exactly the same way as for eqs. (3.6) and (3.7). In the present case, the appropriate analytical continuation, analogous to eq. (3.14), yields:

$$-\frac{d}{ds}F(2s-4)\Big|_{s=0} = \frac{15\,\pi^4}{8}\,\zeta'(-4) + 4\int_0^\infty dy\, y^3 \log\frac{\tilde{c}_i L^{-1} y \tanh y + 1}{\tilde{c}_i L^{-1} y + 1}\,. \tag{3.21}$$

The zeros of the numerator inside the logarithm are those of eqs. (3.19) and (3.20), while the first term subtracts the contributions from the poles of the tangent. Individually, each of these expressions produces a maximum for the radion ϕ (remember that $c_i \sim \phi^{-1/3}$). This can be understood directly from eqs. (3.19) and (3.20). We note that as $\phi \to \infty$, the spectrum is given by $\tilde{k}L = (n+1/2)\pi$, i.e. for large radius the brane terms do not affect the spectrum of an antiperiodic field. In particular, for large ϕ the potential is repulsive. On the other end, for $\phi \to 0$ the spectrum is given by $\tilde{k}L = n\pi$, i.e. as for a field with periodic boundary conditions, which produces an attractive potential. Therefore, the potential must have a maximum at some finite ϕ.[10]

3.1 Exact tree-level propagator including brane-localized terms

In the previous section, we showed that brane-localized terms can have an important effect on the Casimir energy and actually produce a stable minimum for the radion potential. This is not completely surprising since the coefficients of these terms introduce in general a new length scale in the problem. We showed that the inclusion of brane kinetic terms can stabilize the radion at a scale of the order of $c_{0,L}$ of eq. (3.1). Since these coefficients have dimension of length, and since we would like to stabilize the radius at a value somewhat larger than the inverse cutoff of the theory,

[10] This behavior can be confirmed numerically from eq. (3.21); note that the integrand in (3.21), after conveniently choosing $L = c_i$, depends only on ϕ.

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it follows that the coefficients of the brane-localized kinetic terms need to be larger than the fundamental length scale if they are to be relevant for radion stabilization. In particular, their effects should be treated exactly, as we did in the calculation of the Casimir energy.[11]

Then the question arises as to whether it is consistent to allow these terms to be large, while treating other effects perturbatively. In this section, we derive the exact tree-level propagator including brane kinetic terms and argue that perturbation theory does not break down, even if the quadratic brane operators have anomalously large coefficients.

To this end, consider a real scalar field in 5-d flat spacetime with action:

$$S = \frac{1}{2}\int d^4x \int_{-L}^{L} dy \left[\partial_M \Phi \partial^M \Phi + 2c_0 \delta(y) \partial_\mu \Phi \partial^\mu \Phi + 2c_L \delta(y-L) \partial_\mu \Phi \partial^\mu \Phi\right] . \qquad (3.22)$$

In this section, $M = 0,1,2,3,y$, and $\mu = 0,1,2,3$. We assume that the fifth dimension is compactified on S^1 and, to simplify notation, neglect the dependence on ϕ, which can trivially be included. The propagator obeys:

$$\left(\partial_y^2 + p^2 + 2c_0 p^2 \delta(y) + 2c_L p^2 \delta(y-L)\right) G(p;y,y') = \delta(y-y') \,, \qquad (3.23)$$

where we Fourier transformed in the four noncompact coordinates. Since $G(p;y,y')$ is periodic in y with period $2L$, we can write:

$$\begin{aligned} G(p;y,y') &= \frac{1}{2L}\sum_n e^{-i\frac{\pi n}{L}y} G_n(p;y') \,, \\ \delta(y-y') &= \frac{1}{2L}\sum_n e^{-i\frac{\pi n}{L}(y-y')} \,. \end{aligned} \qquad (3.24)$$

Replacing these expansions back in eq. (3.23), dividing by $p^2 - (\pi n/L)^2$ and summing $\sum_n e^{-i\frac{\pi n}{L}y}$, we obtain:

$$G(p;y,y') + 2c_0 p^2 G(p;0,y') B(p,y) + 2c_1 p^2 G(p;L,y') B(p,y-L) = B(p,y-y') \,, \qquad (3.25)$$

where we defined:

$$B(p,y) = \frac{1}{2L}\sum_n \frac{e^{-i\pi n/L y}}{p^2 - (\pi n/L)^2} \,. \qquad (3.26)$$

Evaluating eq. (3.25) at $y = 0$ and $y = L$ gives two equations, from which $G(p;0,y')$ and $G(p;L,y')$ can be found as a function of $B(p,y)$. Eq. (3.25) then gives $G(p;y,y')$. So, it remains only to evaluate the sum in (3.26). This can be done

[11] One way localized kinetic terms could arise in, e.g. a string construction would be through the expectation values of fields, confined to propagate to the fixed points of the orbifolds ("twisted sector" fields); a similar mechanism has been exploited in a field theory context in [37].

by methods similar to those used to evaluate the sums for the Casimir energy (see appendix) and we shall not repeat them here. The result is:

$$B(p, y) = \frac{1}{2p} \csc(pL) \cos\left(\left(1 - \frac{y}{L} + 2\left[\frac{y}{2L}\right]\right) pL\right), \tag{3.27}$$

where $[x]$ denotes the integer closest to x, but smaller than x.

The propagator can then be written as follows:

$$G(p; y, y') = G_+(p; y, y') + G_-(p; y, y'), \tag{3.28}$$

where $G_+(p; y, y')$ is the propagator for the even modes of the field and $G_-(p; y, y')$ is the propagator for the odd modes, that is (denoting $\Phi_\pm(p, y) = (\Phi(p, y) \pm \pm\Phi(p, -y))/2$):

$$\begin{aligned} G_\pm(p; y, y') &= \langle \Phi_\pm(p, y) \Phi_\pm(-p, y') \rangle \\ &= \frac{1}{4} \left(G(p; y, y') \pm G(p; -y, y') \pm G(p; y, -y') + G(p; y, y') \right). \end{aligned} \tag{3.29}$$

The final expression for the propagators of the even and odd modes is easily obtained from eqs. (3.25) and (3.27). In the region $y, y' \in [0, L]$, the propagator of the even modes of the field is:

$$G_+^{(1)}(p; y, y') = -\frac{[\cos(py_<) - c_0 p \sin(py_<)]\,[\cos(p(L - y_>)) - c_L p \sin(p(L - y_>))]}{2p\,[(c_0 c_L p^2 - 1)\sin(pL) - (c_0 + c_L) p \cos(pL)]}, \tag{3.30}$$

where $y_<$ ($y_>$) denotes the smaller (larger) of y, y' and $p \equiv \sqrt{p^2}$. We observe that $G_+(p; y, y')$ has poles whenever $(c_0 c_L p^2 - 1)\sin(pL) - (c_0 + c_L) p \cos(pL) = 0$. This agrees with our result from section 3, see eq. (3.3). In the same region $y, y' \in [0, L]$, the propagator for the odd modes has the much simpler form:

$$G_-^{(1)}(p; y, y') = -\frac{1}{2p} \csc(pL) \sin(py_<) \sin(p(L - y_>)). \tag{3.31}$$

We note that $G_-(p; y, y')$ has poles at $p = n\pi/L$, which is consistent with the fact that the odd modes vanish at the location of the branes and therefore do not feel the localized kinetic terms. The orbifold projection amounts to keeping only $G_+(p; 0, 0)$ for even parity fields and $G_-(p; 0, 0)$ for odd parity fields.

The propagators in the other regions can be easily found from the reflection properties of $\Phi_\pm(p, y)$ in y according to eq. (3.29). Explicitly, when $y, y' \in [-L, 0]$,

$$G_\pm^{(2)}(p; y, y') = G_\pm^{(1)}(p; -y, -y'), \tag{3.32}$$

while for $y \in [0, L]$ and $y' \in [-L, 0]$ or v.v.,

$$G_\pm^{(3)}(p; y, y') = \pm G_\pm^{(1)}(p; |y|, |y'|). \tag{3.33}$$

For y, y' outside the $[-L, L]$ interval, $G_\pm(p; y, y')$ are extended periodically in both y and y'.

We also note that $G_+^{(1)}(p;y,y') \to G_-^{(1)}(p;y,y')$ as both $c_0, c_L \to \infty$. It is easy to check, that in this limit the full propagator $G(p;y,y')$ is precisely the propagator with Dirichlet boundary conditions. This can also be understood by considering the eigenmodes of the scalar field equation and noting that both the even and odd modes vanish at the branes when $c_0, c_L \to \infty$. Thus, the limit of very large kinetic terms reduces to a problem where the branes act as perfect mirrors.

We are now ready to address the main goal of this section — analyzing the divergences occurring in the perturbative expansion when the quadratic brane-localized terms are large, while interaction terms are considered as perturbations. The expressions (3.30) and (3.31) allow us to study the behavior of the propagator as a function of the 4-d momentum p and thus analyze the divergences that will appear in loops with this propagator.

We find, that if $y \neq y'$ the propagator vanishes exponentially for large Euclidean momenta, $p \gg L^{-1}$. This shows that the possible divergences in the theory are local in y, as expected. When $y = y'$ there is a difference whether we are sitting inside the bulk or at one of the branes. In the former case, $G(p;y,y) \sim \frac{1}{2p}$ which corresponds to a 5-d behavior. In the latter case, when $y = y'$ at one of the branes, say $y = 0$, $G(p;0,0) = G_+(p;0,0) \sim \frac{1}{2p+2c_0p^2}$ with c_L in place of c_0 when $y = y' = L$. This is precisely the behavior found in ref. [35] in the case of gravity.[12] On the other end, for $p \ll L^{-1}$, the propagator behaves like $G(p;y,y') \sim ((2c_0 + 2c_L + L)p^2)^{-1}$, which shows the 4-d behavior expected from the finite size of the extra dimension.

If we add interactions to the theory, we can proceed to calculate Feynman diagrams with our exact tree-level propagator, eq. (3.28), or the appropriate one in an orbifolded theory. We imagine that the theory is valid below some scale Λ, and that the 4-d momentum integrals are cutoff at Λ. We also assume that the radius has been stabilized at $L \gg \Lambda^{-1}$, so that there is a region of energies where the 5-d effective theory is valid. The asymptotic behavior of the propagator for large Euclidean momenta, discussed in the previous paragraph, then shows that, in the bulk, the possible divergences of Feynman diagrams are the same as in an uncompactified 5-d theory, independent of the size of the brane kinetic terms. The convergence properties of Feynman diagrams at the branes, however, depend on the size of c_0, c_L. For definiteness, let us concentrate on the $y = 0$ brane and write $c_0 = \hat{c}\Lambda^{-1}$. If the brane couplings are perturbative, i.e. $\hat{c} \lesssim 1$, the high energy behavior of the propagator ($\sim 1/p$) is as expected in a 5-d theory. If, on the other hand $\hat{c} \gg 1$, the convergence properties are improved — they are as in a 4-d theory — since for $p \gg c_0^{-1} = \Lambda/\hat{c}$, the propagator behaves like $\sim 1/p^2$. Thus, it is consistent to assume that $c_i \gg \Lambda^{-1}$,

[12] The models of [35] do not assume that the extra dimensions are compactified. Their point is that the above 4-d behavior due to the brane kinetic terms can make the compactification unnecessary. The fact that our result reduces precisely to theirs in the large momentum region is clear, since we do not expect the large energy behavior to be sensitive to what may happen far away from the brane in question.

in which case the brane kinetic terms can play a role in the stabilization mechanism for the radion. If one is not interested in obtaining a large hierarchy, moderately large c_i might be enough. We remark that, as shown in ref. [33], the presence of brane kinetic terms seems to be quite generic in theories with orbifold fixed points.

To conclude this section, we note that it is straightforward to include bulk and brane mass terms as well. Consider adding to the action of eq. (3.22) the terms:

$$\Delta S = -\frac{1}{2}\int d^4x \int_{-L}^{L} dy \left[M^2 + 2m_0\delta(y) + 2m_1\delta(y-L)\right]\Phi^2\,. \tag{3.34}$$

The analogs of eqs. (3.30) and (3.31) for the propagator derived from $S+\Delta S$ are:

$$\begin{aligned} G_+^{(1)}(p;y,y') &= -\frac{[\cos(\rho y_<) - b_0\rho^{-1}\sin(\rho y_<)]\,[\cos(\rho(L-y_>)) - b_L\rho^{-1}\sin(\rho(L-y_>))]}{2\rho\,[(b_0 b_L\rho^{-2}-1)\sin(\rho L) - (b_0+b_L)\rho^{-1}\cos(\rho L)]}\,, \\ G_-^{(1)}(p;y,y') &= -\frac{1}{2\rho}\csc(\rho L)\sin(\rho y_<)\,,\sin(\rho(L-y_>))\,, \end{aligned} \tag{3.35}$$

where now $b_i = c_i p^2 - m_i$ and $\rho = \sqrt{p^2 - M^2}$. These terms also induce a nontrivial radion potential.

4. Casimir energies in six dimensions compactified on a torus

Now we consider the issue of the Casimir energy and radius stabilization by quantum effects in the richer case of a 6-d spacetime. We consider compactifying two of the dimensions on a torus, where the calculations run in parallel with our 5-d analysis. The toroidal compactification is the one employed in the composite-Higgs models of ref. [23]. The detailed spectrum and current experimental constraints on the size of the $T^2/\mathbb{Z}_2$ orbifold from precision electroweak observables are discussed in ref. [44]. Additional motivation for considering orbifold compactifications of this type is the observation [45] that in 6-d standard-model global anomaly cancellation places severe constraints on the number of identical generations, requiring $n_g = 0 \bmod 3$.

The main difference from the 5-d Casimir energy is that there are now three moduli, which would have to be stabilized. As we will see, the dependence of the Casimir energies on these moduli is nontrivial. To define a torus, one specifies a lattice in the plane and identifies points that differ by a lattice vector. The three moduli can be identified with the lengths of the two lattice vectors and the angle between them. Alternatively, we can use the area of the torus and the modular parameter $\tau = \tau_1 + i\tau_2$. We consider the background interval:

$$ds^2 = \mathcal{A}^{-1}\,\eta_{\mu\nu}dx^\mu dx^\nu + \mathcal{A}\,\gamma_{ij}dy^i dy^j\,, \tag{4.1}$$

where $y^i \in [0,L]$ and the metric γ_{ij} on the torus is:

$$\gamma_{ij} = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & |\tau|^2 \end{pmatrix}. \tag{4.2}$$

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In this parameterization, $det(\gamma) = 1$, so $\mathcal{A}$ is indeed the (dimensionless) area of the torus. Further, the parameterization is such that for fluctuations of the metric with $\eta_{\mu\nu} \to g_{\mu\nu}(x^\mu)$, $\mathcal{A}(x^\mu)$, $\tau(x^\mu)$ depending only on the four noncompact coordinates, we have:

$$M_6^4 \int d^6x \sqrt{G} R(G) = M^2 \int d^4x \sqrt{g} \left[R(g) + \frac{g^{\mu\nu}\partial_\mu \tau \partial_\nu \bar{\tau}}{2\tau_2^2} + \frac{g^{\mu\nu}\partial_\mu \mathcal{A} \partial_\nu \mathcal{A}}{\mathcal{A}^2} \right], \qquad (4.3)$$

where $M^2 = L^2 M_6^4$,[13] R is the Ricci scalar evaluated with the metric indicated in parenthesis and $\bar{\tau} \equiv \tau_1 - i\tau_2$. There are also two massless spin-one fields with their corresponding Kaluza-Klein towers. These modes also contribute to the total Casimir energy, but they do not play a role in the calculation of the Casimir energy itself (see below) so we have not written them explicitly. We will see below that their zero modes are projected out in the orbifold.

As in the 5-d case, it is enough to calculate the Casimir energy of a real scalar field in the desired background (see discussion of gauge fixing below). Thus, we begin with the vacuum energy of a single massless scalar field propagating in the background (4.1), with periodic boundary conditions:

$$\begin{aligned} V^{+,\,\mathrm{scalar}} &= \frac{1}{2} \sum_{m,n=-\infty}^{\infty} \int \frac{d^4k}{(2\pi)^4} \log\left(R^2 k^2 + \mathcal{A}^{-2}\tau_2^{-1}|m - n\tau|^2\right) \\ &= -\frac{d}{ds}\zeta^{+,\,\mathrm{scalar}}(s)|_{s=0}\,, \end{aligned} \qquad (4.4)$$

where $R \equiv L/(2\pi)$. Here we used again ζ-function regularization to define this expression, where now:

$$\begin{aligned} \zeta^{+,\,\mathrm{scalar}}(s) &= \frac{1}{2} {\sum_{m,n}}' \int \frac{d^4k}{(2\pi)^4} \left(R^2 k^2 + \mathcal{A}^{-2}\tau_2^{-1}|m - n\tau|^2\right)^{-s} \\ &= \frac{\pi^2}{L^4} \frac{\mathcal{A}^{2s-4}}{(s-2)(s-1)} {\sum_{m,n}}' \frac{\tau_2^{s-2}}{|m - n\tau|^{2s-4}}\,, \end{aligned} \qquad (4.5)$$

and the prime indicates that the zero mode is to be excluded. It is clear from the second line that $\zeta^{+,\,\mathrm{scalar}}(s)$ is modular invariant, reflecting the fact that the Casimir energy is independent of the discrete choice of lattice vectors defining the torus (these choices are related by $SL(2,\mathbb{Z})$ transformations of the modular parameter τ, see, e.g. [46]). We defer the evaluation of the sum in (4.5) to the Appendix. The result for the potential is:

$$V^{+,\,\mathrm{scalar}} = -\frac{\pi^2}{2L^4} \mathcal{A}^{-4} f(\tau, \bar{\tau})\,, \qquad (4.6)$$

[13] M is not yet the physical 4-d Planck mass that characterizes the gravitational interactions. Once $\mathcal{A}$ gets a vacuum expectation value, it will be convenient to do a constant Weyl rescaling $\tilde{g}_{\mu\nu} = \mathcal{A}^{-1} g_{\mu\nu}$. It is $\tilde{g}$ that couples to all fields. As a result, it is the coordinate independent combination $\mathcal{A}L^2$ that appears everywhere and the physical 4-d Planck mass is $M_4^2 = (\mathcal{A}L^2)M_6^4$.

with (taking $\tau_2 > 0$ without loss of generality):

$$f(\tau,\bar\tau) = \frac{8\pi}{945}\tau_2^3 + \frac{3\zeta(5)}{\pi^4}\frac{1}{\tau_2^2} + \frac{4}{\pi^2}\sum_{m=1}^{\infty}\left(m^2\,\mathrm{Li}_3(q^m) + \frac{3}{2\pi}\frac{1}{\tau_2}\,m\,\mathrm{Li}_4(q^m) + \right.$$
$$\left. + \frac{3}{4\pi^2}\frac{1}{\tau_2^2}\,\mathrm{Li}_5(q^m) + \mathrm{h.c.}\right), \qquad (4.7)$$

where $\mathrm{Li}_n(x)$ are the polylogarithm functions and $q = e^{2\pi i(\tau_1 + i\tau_2)}$.

A few comments are due on the properties of the function $f(\tau,\bar\tau)$ in (4.6). As mentioned above, $f(\tau,\bar\tau)$ is invariant under $\mathrm{SL}(2,\mathbb{Z})$ modular transformations ($\tau \to (a\tau+b)/(c\tau+d)$, where a, b, c, d are integers, obeying $ad - bc = 1$) due to the fact that tori with modular parameters related by an $\mathrm{SL}(2,\mathbb{Z})$ transformation are diffeomorphic. Distinct tori have modular parameters τ taking values in the fundamental region $|\tau| \geq 1$, $1/2 > \tau_1 \geq -1/2$, $\tau_2 > 0$. It is therefore enough to study the behavior of the Casimir energy (4.6) in the fundamental region. The points $\tau = i$ and $\tau = e^{2\pi i/3}$ of the fundamental region are fixed points of the transformations $\tau \to -1/\tau$ and $\tau \to -1/(1+\tau)$, respectively, and are thus extrema of $f(\tau,\bar\tau)$. Numerically we find that the point $\tau = i$, corresponding to a rectangular torus, is a saddle point, the point $\tau = e^{2\pi i/3}$, corresponding to a torus of angle 120 degrees, is a minimum, and that there are no other extrema of $f(\tau,\bar\tau)$. The function $f(\tau,\bar\tau)$ is positive definite and unbounded as $\tau_2 \to \infty$ in the fundamental region. Therefore, regarding the dependence of the Casimir energy on the modular parameter τ, for fixed area $\mathcal{A}$, we can conclude the following: i) if the Casimir energy is attractive, as for a periodic scalar, eq. (4.6), the energy is unbounded below as $\tau_2 \to \infty$. ii) if the Casimir energy is repulsive, the torus with $\tau = e^{2\pi i/3}$ minimizes the potential for a fixed area. For repulsive total Casimir energy, therefore, stabilizing the area modulus leads to stabilization of the modular parameter.

To calculate now the total contribution to the Casimir energy due to the gravitational fluctuations, it is necessary to count the number of physical degrees of freedom. We write the 6-d metric as $G_{MN} = G^{(0)}_{MN} + h_{MN}$, where $G^{(0)}_{MN}$ is the background defined in eq. (4.1) and h_{MN} are the quantum fluctuations. To begin, note that in a flat background it is always possible to fix a "transverse-traceless" gauge:

$$G^{(0)MK}\partial_K \bar h_{MN} = 0$$
$$\bar h = 0$$

where $\bar h_{MN} = h_{MN} - 1/2 G^{(0)}_{MN} h$ and $\bar h = G^{(0)MN}\bar h_{MN}$. This gauge fixing conditions eliminate 7 of the original 21 polarizations. The fluctuations then satisfy the wave equation $G^{(0)KL}\partial_K\partial_L h_{MN} = 0$. However, there is still a residual gauge freedom parameterized by $h'_{MN} = h_{MN} + \partial_M \xi_N + \partial_N \xi_M$, where $G^{(0)KL}\partial_K\partial_L \xi_M = 0$. Working in Fourier space (discrete along the two compactified dimensions), it is easy to see that, for modes with a nonzero momentum in the compact dimensions, there is enough freedom to set:

$$
\begin{aligned}
h_{\mu 6}(x, y^1, y^2) &= -h_{\mu 5}(x, y^1, y^2) \\
h_{55}(x, y^1, y^2) &= h_{66}(x, y^1, y^2)\,.
\end{aligned}
$$

So the massive modes (from a 4-d perspective) include a tower of spin-2 fields (with 5 polarizations each), a tower of spin-1 fields (with 3 polarizations each) and a tower of real scalars. All of these give a contribution to the Casimir energy which is 9 times that of eq. (4.6).

The zero modes include (before orbifolding) the 4-d graviton, two gauge fields, and the three moduli that define the size and shape of the torus; as usual, the zero modes do not contribute to the Casimir energy. We also quote the contributions of other relevant bulk fields:

$$
\begin{aligned}
V^{+,\,\mathrm{graviton}} &= 9\,V^{+,\,\mathrm{scalar}}\,, \\
V^{+,\,\mathrm{Weylfermion}} &= -4\,V^{+,\,\mathrm{scalar}}\,, \\
V^{+,\,\mathrm{vector}} &= 4\,V^{+,\,\mathrm{scalar}}\,, \\
V^{+,2form} &= 3\,V^{+,\,\mathrm{scalar}}\,, \\
V^{+,gravitino} &= -12\,V^{+,\,\mathrm{scalar}}\,.
\end{aligned}
\tag{4.8}
$$

The simplest way to arrive at these results is by knowing the 6-d $(1,0)$ supersymmetry multiplets. The toroidal compactification preserves supersymmetry, hence the Casimir energy should vanish within supermultiplets. The second equation follows from the fact that the $(1,0)$ hypermultiplet contains a complex Weyl fermion and two complex scalars, while the third follows from the content of a $(1,0)$ vector multiplet (a vector field and a Weyl fermion). The self-dual two form field contribution is denoted by $V^{+,2form}$ (recall that a $(1,0)$ tensor multiplet consists of a Weyl fermion, an anti-self-dual two form antisymmetric tensor field, and a real scalar). Finally, the last line follows from the structure of the $(1,0)$ gravity multiplet: graviton, gravitino, and a self-dual two form.

Note that if we orbifold the torus by a discrete subgroup of the 2-d rotation group, it is possible to "freeze" the values of τ_1 and τ_2 and concentrate on the $\mathcal{A}$ dependence of (4.6) alone.[14] For example, if we write the coordinates in the extra dimensions in complex notation as $z = y^1 + iy^2$ and identify points $z \sim e^{2\pi i/n} z$, it is easy to see that whenever $n > 2$, the requirement that the lattice that defines the torus have the appropriate symmetry implies that τ_1 and τ_2 must take on particular values.[15]

[14] This would probably be necessary in any case to obtain chiral fermions in the low energy theory.

[15] In particular, the case $n = 4$ corresponds to a square torus with $\tau_1 = 0$, $\tau_2 = 1$. In this case, the sum $\sum{}'(n^2+m^2)^{-s}$ in eq. (4.5) can be evaluated more easily with the help of the Jacobi identities. We have checked that eq. (4.7) reproduces this result.

The orbifolding will also project out some of the modes of the gravity multiplet as can be seen by the requirement that the line element be invariant. For example, the case $n = 4$ corresponds to the identifications $(y^1, y^2) \sim (-y^2, y^1)$. Writing the line element as:

$$ds^2 = G_{\mu\nu}dx^\mu dx^\nu + A_{\mu i}dx^\mu dy^i + \gamma_{ij}dy^i dy^j\,, \tag{4.9}$$

determines the transformation properties of the various metric components:

$$\begin{aligned}
G_{\mu\nu}(x^\mu, -y^2, y^1) &= G_{\mu\nu}(x^\mu, y^1, y^2)\\
A_{\mu 1}(x^\mu, -y^2, y^1) &= -A_{\mu 2}(x^\mu, y^1, y^2)\\
A_{\mu 2}(x^\mu, -y^2, y^1) &= A_{\mu 1}(x^\mu, y^1, y^2)\\
\gamma_{11}(x^\mu, -y^2, y^1) &= \gamma_{22}(x^\mu, y^1, y^2)\\
\gamma_{12}(x^\mu, -y^2, y^1) &= -\gamma_{21}(x^\mu, y^1 . y^2)\,.
\end{aligned} \tag{4.10}$$

These parity assignments imply $A_{\mu i}(x^\mu, -y^1, -y^2) = -A_{\mu i}(x^\mu, y^1, y^2)$ and $\gamma_{12}(x^\mu, -y^1, -y^2) = -\gamma_{12}(x^\mu, y^1, y^2)$. So the zero modes of these components are projected out. Also all their Kaluza-Klein components are forced to vanish at the positions of the orbifold fixed points $(y^1, y^2) = (0, 0), (0, L), (L, 0)$ and (L, L). In addition, the other (y-independent) zero modes (apart from the 4-d graviton) satisfy $\gamma_{11}(x^\mu) = \gamma_{22}(x^\mu)$, so there is only one modulus left, which can be identified with the area of the torus.

Quite generally, there will be a discrete set of points that are left invariant by the orbifold identification. As discussed in the previous sections, we expect the presence of terms in the action localized on these fixed points. Here we consider only the effect of the brane tensions, which gives a contribution to the potential for the area modulus $\mathcal{A}$:

$$T \int d^6x\, \delta(\vec{y})\, \sqrt{\tilde{g}} = T \int d^4x\, \mathcal{A}^{-2}\,, \tag{4.11}$$

where $\tilde{g}$ is the induced metric at the relevant fixed point and T — the corresponding brane tension. Adding these terms, as well as a bulk cosmological constant:

$$\Lambda \int d^6x\, \sqrt{G} = \Lambda L^2 \int d^4x \mathcal{A}^{-1}\,, \tag{4.12}$$

to the contributions of the massless fields (4.6) it is possible to obtain a minimum for $\mathcal{A}$ that satisfies the normalization conditions (2.4), as in the 5-d case.

If we sum the contributions of all fields to the Casimir energy in the model of ref. [23], we find that the potential for the area modulus is repulsive. We note that ref. [23] used a $\mathbb{Z}_2$ orbifold, in which case there are two noncompact moduli; it is¡, however, possible to generalize the construction to higher $\mathbb{Z}_n$'s, affecting only the massive spectrum, and thus project all but the area modulus (alternatively, since the Casimir energy in the model with minimal matter content is repulsive, as remarked after eq. (4.6), for fixed area the torus with $\tau = e^{2\pi i/3}$ has minimal

Casimir energy). With just the area modulus, a minimum with zero cosmological constant can be obtained after fine tuning the coefficients of the counterterms (4.11), and (4.12).

5. Applications

In this section, we consider the question of radius stabilization in the supersymmetric 5-d model of [22] (see also [23]). This model is highly predictive, and the authors of ref. [22] were able to determine not only the Higgs and superpartner masses, but also the compactification radius of the fifth dimension, which, in fact, sets the scale for all these masses. However, they did not provide a stabilization mechanism for the radius. We are interested in determining whether the radius can be stabilized by quantum effects due to the original (minimal) field content.

Let us briefly review the main elements of the model of ref. [22]. One starts from a five dimensional setup where the standard model superfields propagate in the bulk. The fifth dimension is compactified on S^1, which is then orbifolded by identifying points that are related by two $\mathbb{Z}_2$ parities. The first one gives just the $S^1/\mathbb{Z}_2$ orbifold studied in section 2 and breaks half the supersymmetry. The second identification breaks the rest of the supersymmetry giving precisely the standard model at low energies (below the compactification scale).

The standard model gauge fields are described by 5-d vector supermultiplets $(A^M, \lambda, \lambda', \sigma)$, which from a 4-d point of view decompose into one $N = 1$ vector supermultiplet $V = (A^\mu, \lambda)$ and one chiral superfield in the adjoint representation $\Sigma(\phi_\Sigma, \psi_\Sigma)$, where $\Phi_\Sigma = (\sigma + iA^5)/\sqrt{2}$. The standard model matter and Higgs fields are described by hypermultiplets $(\Psi, \Phi, \Phi')_X$, where $X = M$ $(= Q, U, D, L, E)$ for matter fields and $X = H$ for the single Higgs field of the model. Here Ψ is a Dirac fermion and Φ, Φ' are two complex scalars. In 4-d language, these correspond to two $N = 1$ chiral supermultiplets $X(\Phi_X, \psi_X)$ and $X^c(\Phi^c_X, \psi^c_X)$, where $\Psi = (\psi_X, \psi^c_X)$ and $\Phi' = \Phi^{c\dagger}$.

In this model, matter and Higgs superfields are distinguished by different parity assignments under the $\mathbb{Z}_2 \times \mathbb{Z}'_2$ projection. An equivalent way to describe this projection, which is more suited to the notation employed in section 2.3 is the following: we start from the $S^1/\mathbb{Z}_2$ orbifold discussed in section 2 and break the remaining 4-d $N = 1$ supersymmetry by the Scherk-Schwarz mechanism, imposing

	$(+,+)$	$(+,-)$	$(-,+)$	$(-,-)$
M	ψ_M	Φ_M	$\psi^{c\dagger}_M$	$\Phi^{c\dagger}_M$
H	Φ_H	ψ_H	$\Phi^{c\dagger}_H$	$\psi^{c\dagger}_H$
V	A_μ	λ	ϕ_Σ	ψ_Σ

Table 1: Parity assignments for matter, Higgs and vector fields in the model of ref. [22]. The first assignment refers to the $\mathbb{Z}_2$ parity and the second to the R_P parity of each component field.

$$\Phi(y + L) = R_P \Phi(y)\,, \tag{5.1}$$

where R_P is the R-parity of the unbroken supersymmetry of $S^1/\mathbb{Z}_2$ and Φ represents any component field. Eq. (5.1) is the same as imposing periodic (antiperiodic) boundary conditions on the fundamental interval $[0, L]$ on the fields with R_P even (odd). If we denote the possible parity assignments by $(+,+)$, $(+,-)$, $(-,+)$, $(-,-)$ where the first entry refers to the $\mathbb{Z}_2$ and the second to the R_P parity, the component field parity assignments are as given in table 1.

In addition, one can write supersymmetric interactions at the orbifold fixed points (using the notation of ref. [22]):

$$\Delta L = \frac{1}{2}(\delta(y) + \delta(y-2L)) \int d^2\theta(\lambda_U QUH) + \frac{1}{2}(\delta(y-L) + \delta(y+L)) \times \\ \times \int d^2\theta'(\lambda_D Q'D'H' + \lambda_E L'E'H') + \text{h.c.}\,, \tag{5.2}$$

which give rise to standard model quark masses after electroweak symmetry breaking.

Ref. [22] calculated the one-loop effective potential for the standard model Higgs taking into account the contribution of the top Yukawa coupling (the other Yukawa couplings are much smaller and are neglected). We note that this is precisely the Casimir energy due to the Q and U matter superfields in the presence of brane masses ($= \lambda_U\langle H\rangle$). Adding the tree-level potential (from the D-terms) to this quantum contribution, the authors of ref. [22] found a Higgs potential of the form (see eqs. (15) and (31) of ref. [22]):

$$V_{BHN}(\phi, H) = \frac{1}{\phi^2 L^4}\hat{V}(\phi^{1/3}LH)\,, \tag{5.3}$$

where ϕ is the radion field in the parameterization eq. (2.1). The minimization of this potential with respect to H fixes the combination $x \equiv \phi^{1/3}LH$ at a value x_0 that can be determined numerically. Knowledge of the Higgs vev then determines the size of the extra space $\phi^{1/3}L$. However, eq. (5.3) is only part of the contribution to the potential for ϕ. As discussed in section 2, all other fields, as well as possible cosmological and brane tensions contribute to $V(\phi)$.

To analyze this potential, we first neglect the possible contributions from brane kinetic terms. Then the spectrum for each of the parity assignments is:

$$\begin{aligned} m_n^{(+,+)} &= 2n\pi/L \\ m_n^{(+,-)} &= (2n+1)\pi/L \\ m_n^{(-,+)} &= (2n+2)\pi/L \\ m_n^{(-,-)} &= (2n+1)\pi/L \end{aligned} \tag{5.4}$$

with $n = 0, 1, 2, \ldots$ We can obtain the Casimir energies in the present orbifold from eq. (2.10) by making the replacement $L \to L/2$ and including a factor of $\frac{1}{2}$ due to the first $\mathbb{Z}_2$ orbifold, thus giving a total factor of 8. Thus, the contribution from the

KK tower of each real degree of freedom is:

$$\begin{aligned} V^{(+,+),\,\mathrm{realscalar}}(\phi) &= 8V^{+,\,\mathrm{scalar}}(\phi) \\ V^{(+,-),\,\mathrm{realscalar}}(\phi) &= 8V^{-,\,\mathrm{scalar}}(\phi) \\ V^{(-,+),\,\mathrm{realscalar}}(\phi) &= 8V^{+,\,\mathrm{scalar}}(\phi) \\ V^{(-,-),\,\mathrm{realscalar}}(\phi) &= 8V^{-,\,\mathrm{scalar}}(\phi)\,, \end{aligned} \tag{5.5}$$

where $V^{+,\mathrm{scalar}}(\phi)$ and $V^{-,\mathrm{scalar}}(\phi)$ are given in eqs. (2.11) and (2.13), respectively. Taking into account the number of degrees of freedom and parity assignments given in table 1, we get for matter, Higgs and vector supermultiplets the following contributions to the scalar potential:

$$\begin{aligned} V^{(M)}(\phi) &= -62V^{+,\,\mathrm{scalar}}(\phi) \\ V^{(H)}(\phi) &= +62V^{+,\,\mathrm{scalar}}(\phi) \\ V^{(V)}(\phi) &= +62V^{+,\,\mathrm{scalar}}(\phi)\,. \end{aligned} \tag{5.6}$$

If we consider now the field content of the present model, namely the $\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ vector multiplets, three generations of matter hypermultiplets (less the third generation Q and U superfields which were included in eq.(5.3)), and one Higgs hypermultiplet, we get a total contribution:

$$V^{\mathrm{massless}}(\phi) = -1364V^{+,\,\mathrm{scalar}}(\phi) = +\frac{1023\zeta(5)}{16}\frac{1}{\phi^2 L^4}\,. \tag{5.7}$$

Thus the total radion potential is:

$$L^4 V(\phi) = \alpha\phi^{-1/3} + \beta\phi^{-2/3} + \gamma\phi^{-2}\,, \tag{5.8}$$

where $\gamma \equiv \hat{V}(x) + 1023\zeta(5)/16$ includes the contributions from all matter fields and we also included a bulk cosmological constant α, and possible brane tensions β. As we said before, the minimization with respect to H, $\hat{V}'(x) = 0$, fixes $x = \phi^{1/3}LH$ at some x_0 and also the value of $\hat{V}_0 \equiv \hat{V}(x_0)$ in eq.(5.3) (which turns out to be positive). The equation of motion for ϕ then requires:

$$-\phi^{-1}\left(\frac{\alpha}{3}\phi^{-1/3} + \frac{2\beta}{3}\phi^{-2/3} + 2\gamma\phi^{-2}\right) = 0\,. \tag{5.9}$$

In addition, the requirement that the cosmological constant vanishes implies:

$$\alpha\phi^{-1/3} + \beta\phi^{-2/3} + \gamma\phi^{-2} = 0\,. \tag{5.10}$$

We note that solving eqs.(5.9) and (5.10) imposes a fine-tuning condition among α, β and γ. This is of course the cosmological constant problem, which we are not trying to address here. Note also that we found that $\gamma > 0$, which is a consequence of the fact

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that the matter content — which produces a repulsive potential — dominates over the gauge and Higgs contributions.[16] We find then, that eqs. (5.9) and (5.10) have a solution — which is a minimum — provided $\alpha > 0$ and $\beta < 0$ (and the fine-tuning condition is satisfied). Thus, this mechanism could only stabilize the radius if the bulk cosmological constant was positive (de Sitter) and there are negative tension branes. The first condition seems to be especially problematic, since a positive cosmological constant seems to be incompatible with supersymmetry. We also note that the minimum so obtained is not the only minimum. Since the potential vanishes for large radius, there is a second degenerate minimum at $\phi = \infty$.

It is possible — and we leave this for future[17] study — that the contribution from brane kinetic terms that we discussed in section 3 could change the potential enough to produce a minimum with zero 4-d cosmological constant even when the bulk cosmological constant is negative. However, we note that this would be at best a local minimum, and the true minimum would be anti-de Sitter. To see this, we note that the cosmological constant term always dominates at large separation of the branes (i.e. at large ϕ), so for negative bulk cosmological constant the potential will always tend to zero from below. If there is a minimum at a finite ϕ where the potential vanishes, there will always be a second minimum where the potential is negative. In this case one would have to worry about the tunneling probability to the true AdS vacuum (recall, however, the suppression of tunneling to AdS space [47]).

To conclude this section, an issue which deserves a comment is that of the scale of 5-d gravity (and of 6-d gravity from the previous section) in this model and the consistency of our approximations, which ignored warping due to brane tension and bulk cosmological constant counterterms. This is clearly a concern, since the 5-d gauge theory is non-asymptotically free and the couplings blow up a decade or so above the compactification radius, demanding a transition to a more fundamental description of the theory somewhere in the (multi)-TeV region. If the fundamental scale of 5-d (or 6-d) gravity was in that range as well, we would need a mechanism explaining the smallness of the observed gravitational interaction in four dimensions.

One possibility in the present context is to assume that the higher dimensional gravity scale is in the multi-TeV range while the weakness of 4-d gravity is due to the presence of additional large dimensions (e.g., of millimeter size) accessible only to gravity, in the spirit of [9]. The radius stabilization mechanism is then more complicated due to the presence of these extra dimensions (also, some of our formulae for the Casimir energies would need to be modified). More of a concern

[16]And over the gravity multiplet — it is easy to check that its attractive contribution, neglected in (5.7), is much smaller than the matter fields' repulsion.

[17]It would be also interesting to see to what extent the predictions of the model depend on the values of the brane kinetic terms. In particular, bounds on the coefficients of the kinetic terms from single Kaluza-Klein-mode production should be revisited — as follows from eqs. (3.4), (3.5), the interactions of these modes depend nontrivially on their excitation number.

in this regard is the neglect of the backreaction of brane tensions and cosmological constant counterterms — since their size is determined essentially by the inverse compactification scale (of order TeV), which is not much smaller than the scale of the higher dimensional gravity.

Another possibility — which makes neglecting the backreaction of branes and bulk more palatable — is to assume that the 5-d (or 6-d) gravity scale M is close to the four dimensional observed $M_{\rm Planck}$ and the two are related in the usual way, e.g. $M_{\rm Planck}^2 \sim L_{\rm phys} M^3$ for the 5-d case. From a low-energy point of view this constitutes an unexplained fine-tuning, but may have its origin, as recently pointed out in [48], in "little string theory" (the term first appeared in [49]; for a review, see [50]). A major assumption required in order to embed the models discussed in this paper in this framework is that there is a window of energies above the compactification scale where the field theory description is still valid.

6. Concluding remarks.

This paper was devoted to the Casimir energy in five and six dimensional field theory orbifolds and its effect on radion stabilization. We were motivated by recent models of electroweak symmetry breaking, which used 5-d (supersymmetric) [22] and 6-d [23] orbifolds.

We gave a general discussion of the divergences of the Casimir energy and the counterterms required for their cancellation. We computed the Casimir energy for gravity and various massless fields, obeying different boundary conditions. In the massive case, we pointed out a mechanism for stabilizing compact dimensions, requiring that the massive contribution be repulsive, while the massless be attractive.

We discussed in detail the influence of kinetic terms, localized at the orbifold fixed points, on the Casimir energy. We pointed out that these brane-localized kinetic terms can also generate stable minima for the radion. In the case of localized kinetic terms of size consistent with NDA [36], one can stabilize the compact dimension at a size several times the fundamental length cutoff. For larger brane-localized kinetic terms, that size can be larger. We argued that the low-energy perturbative expansion in theories where brane kinetic terms are larger than the inverse cutoff scale is consistent (in accord with ref. [35]).

Applying our results to the 5-d and 6-d models of electroweak breaking, mentioned above [22, 23], we found that, with the minimal field content, the radion potential generated at one loop due to the Casimir energy is repulsive in both cases. We found that it is possible, by adding brane-tension and cosmological constant counterterms to find a stable minimum for the radion.

However, in the supersymmetric 5-d case [22], the sign of the required bulk cosmological constant turned out to be positive. Thus, deciding whether this fine-tuning is possible in supergravity would need to wait for a full embedding of the model in 5-d supergravity. Alternatively, our result might indicate that a classical mechanism of stabilization is preferred; of course, it would also have to be shown to be consistent with 5-d supergravity.

Finally, we note that, in this paper, we concentrated on static issues, namely the existence of stable minima of the radion field(s). Dynamical issues, such as the evolution of the universe as a backreaction to the Casimir potentials are of interest as well. It would also be extremely interesting to find what radius stabilization mechanism is consistent with supergravity in the supersymmetric case.

Acknowledgments

We thank T. Appelquist, P. Horava, and M. Luty for useful discussions. We also acknowledge support of DOE contract DE-FG02-92ER-40704.

A. Evaluation of Casimir sums

Here, we evaluate some of the sums that appear in the calculation of the Casimir energies in 5- and 6-dimensional models, using ζ-function regularization. Even though the methods for their evaluation are standard, see [46, 51], we show some of the details for completeness. First, we evaluate the single sum:

$$F(s;a,c) \equiv \sum_{n=-\infty}^{\infty} \frac{1}{[(n+a)^2+c^2]^s}\,, \tag{A.1}$$

which is convergent for sufficiently large positive $\mathrm{Re}(s)$. The idea is to recast the sum into a form suitable for analytical continuation to the values of s of interest. Setting $a=0$, this is the sum that appears in the evaluation of the Casimir energy due to a massive scalar field in 5-d, with periodic boundary conditions. For antiperiodic boundary conditions we need $a=1/2$. We start by noting that eq. (A.1) defines a periodic function of a with period 1. So we can expand eq. (A.1) in Fourier series and write:

$$\begin{aligned}
F(s;a,c) &= \sum_p e^{i2\pi pa} \int_0^1 dy e^{-i2\pi py} \sum_n \frac{1}{[(n+y)^2+c^2]^s} \\
&= \sum_p e^{i2\pi pa} \sum_n \int_n^{n+1} dz e^{-i2\pi pz} \frac{1}{[z^2+c^2]^s} \\
&= \sum_p e^{i2\pi pa} \int_{-\infty}^{\infty} dz e^{-i2\pi pz} \frac{1}{\Gamma(s)} \int_0^{\infty} dt\, t^{s-1} e^{-(z^2+c^2)t} \\
&= \frac{\sqrt{\pi}}{\Gamma(s)} |c|^{1-2s} \sum_p e^{i2\pi pa} \int_0^{\infty} du\, u^{s-3/2} e^{-(u+\pi^2p^2c^2u^{-1})}
\end{aligned}$$

$$= \frac{\sqrt{\pi}}{\Gamma(s)}|c|^{1-2s}\Big(\int_0^\infty du\, u^{s-3/2}e^{-u} + \\ + 2\sum_{p=1}^\infty \cos(2\pi pa)\int_0^\infty du u^{s-3/2}e^{-(u+\pi^2p^2c^2u^{-1})}\Big)\,.$$

In the third line we used the representation:

$$z^{-s} = \frac{1}{\Gamma(s)}\int_0^\infty dt\, t^{s-1}e^{-zt}\,, \tag{A.2}$$

while in the fourth line we performed the gaussian integral over z and made the change of variable $u = c^2 t$. In the last line, we recognize the integral representation of the modified Bessel function:

$$K_s(|x|) = 2^{s-1}x^{-s}\int_0^\infty du\, u^{s-1}e^{-(u+x/(4u))} \tag{A.3}$$

which is valid when $Re(x^2) > 0$. Thus, we finally get for (A.1):

$$F(s;a,c) = \frac{\sqrt{\pi}}{\Gamma(s)}|c|^{1-2s}\left(\Gamma\left(s-\frac{1}{2}\right) + 4\sum_{p=1}^\infty (\pi p|c|)^{s-1/2}\cos(2\pi pa)K_{s-1/2}(2\pi p|c|)\right)\,. \tag{A.4}$$

Note that the expression (A.4) is now valid even for negative s. According to eq. (2.15), we need to differentiate eq. (A.4) with respect to s and set $s = -2$. Since $\Gamma(-2) = \infty$, the derivative needs to act only on $\Gamma(s)$ in eq. (A.4). The sum over p, when evaluated at $s = -2$, can be done exactly in terms of the polylogarithm functions

$$\mathrm{Li}_n(x) = \sum_{k=1}^\infty \frac{x^k}{k^n}\,. \tag{A.5}$$

The result is:

$$\frac{d}{ds}F(s;a,c)\Bigg|_{s=-2} = -\frac{16\pi}{15}|c|^5 + \frac{1}{2\pi^4}(4\pi^2|c|^2\mathrm{Li}_3(q) + 6\pi|c|\mathrm{Li}_4(q) + \\ + 3\mathrm{Li}_5(q) + h.c.)\,, \tag{A.6}$$

where we used $\Gamma(5/2) = -8\sqrt{\pi}/15$ and defined $q = e^{2\pi i(a+i|c|)}$. Setting $a = 0$ or $a = 1/2$ (for periodic and antiperiodic boundary conditions respectively) and replacing in eq. (2.15) gives eq. (2.19) of section 2.4 (the first term in eq. (A.6) corresponds to a finite contribution to the bulk cosmological constant and was omitted in eq. (2.4), since other infinite contributions have already been discarded by the regularization procedure).

Next, we turn to the double sum needed in the 6-d models, discussed in section 4:

$$\sum_{m,n}{}' \frac{1}{|n+m\tau|^{2s}}\,, \tag{A.7}$$

where the prime indicates that the zero mode is to be excluded and $\tau = \tau_1 + i\tau_2$. Writing the sum as:

$$\sum_m{}'\sum_n \frac{1}{|n+m\tau|^{2s}} + \sum_n{}' \frac{1}{n^{2s}} = \sum_m{}'\sum_n \frac{1}{[(n+m\tau_1)^2 + m^2\tau_2^2]^s} + 2\zeta(2s)\,, \quad \text{(A.8)}$$

we note that the sum over n in the first term is of the form eq. (A.1), with $a = m\tau_1$ and $c = m\tau_2$. Thus, using eq. (A.4) we obtain:

$$\sum_{m,n}{}' \frac{1}{|n+m\tau|^{2s}} = 2\zeta(2s) + \frac{\sqrt{\pi}\Gamma\left(s-1/2\right)}{\Gamma(s)}|\tau_2|^{1-2s}\, 2\zeta(2s-1) + \\ + \frac{8\pi^s}{\Gamma(s)}|\tau_2|^{1/2-s}\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\left(\frac{p}{m}\right)^{s-1/2}\cos(2\pi p m\tau_1)K_{s-1/2}(2\pi p m|\tau_2|)\,.$$

Differentiating with respect to s, setting $s = -2$ and expressing the sum over p in terms of the polylogarithm functions as we did in eq. (A.6), gives eq. (4.7) of section 4.

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